OCR
RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2601

Pure Mathematics 1

Tuesday **5 JUNE 2001** Morning 1 hour 20 minutes

Additional materials:
- Answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer **all** questions.

You are permitted to use only a scientific calculator for this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

There is an **insert** for use with Question **11.**

The total number of marks for this paper is 60.

This question paper consists of 4 printed pages and an insert.

Section A (30 marks)

1 Find $\int 12x^5\,dx$. [2]

2 Find the roots of the equation $2\sin\theta + 1 = 0$ for which $0° < \theta < 360°$. [3]

3 Solve the inequality $|x-1| < 4$. [3]

4 Find the coefficient of x^6 in the expansion of $(2+x)^{10}$. [3]

5 Prove that the condition for the equation $px^2 - 5x + p = 0$ to have real roots is $-\frac{5}{2} \leqslant p \leqslant \frac{5}{2}$. [3]

6 Express $x^2 + 6x + 1$ in the form $(x+a)^2 + b$.

Find the values of x where the curve $y = x^2 + 6x + 1$ crosses the x-axis. [4]

7 Convert $\frac{1}{6}\pi$ radians to degrees. State the exact value of $\tan\left(\frac{1}{6}\pi\right)$.

Sketch the graph of $y = \tan\theta$ for $0 \leqslant \theta \leqslant 2\pi$. [4]

8 Find in factorised form the cubic polynomial $f(x)$ satisfying the following conditions:

$$f(1) = 0, \quad f(3) = 0, \quad \text{the coefficient of } x^3 \text{ is } 1, \quad \text{and} \quad f(2) = 6.$$ [4]

9 A curve has equation $y = x^3 + x^2 + kx$, where k is a constant. Write down an expression for $\frac{dy}{dx}$.

The curve has a minimum at the point where $x = 1$. Find the value of k.

Find the x-coordinate of the maximum point on the curve. [4]

Section B (30 marks)

10 (i) Show that the line $y = \frac{1}{2}x - \frac{9}{4}$ is the tangent to the curve with equation $y = \frac{1}{4}x^2 - x$ at the point A $(3, -\frac{3}{4})$. [4]

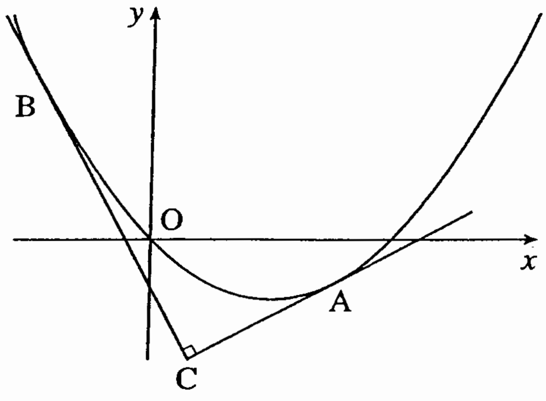

Fig. 10

(ii) The point B on the curve is such that the tangent at B is perpendicular to the tangent at A, as shown in Fig. 10. Find the coordinates of B.

The tangent at A and the tangent at B meet at the point C. Show that triangle ABC has area $\frac{125}{16}$. [10]

[Total: 14]

Question 11 is on the next page

11 A team of historians finds the plans for a Victorian underground water supply tunnel. The cross-section is illustrated in Fig. 11; the units on the graph are yards. The historians wish to estimate the area of the cross-section. The table shows values that were found with the plans.

x	−4	−3	−2	−1	0	1	2	3	4
y	1.00	2.00	2.59	2.90	3.00	2.90	2.59	2.00	1.00

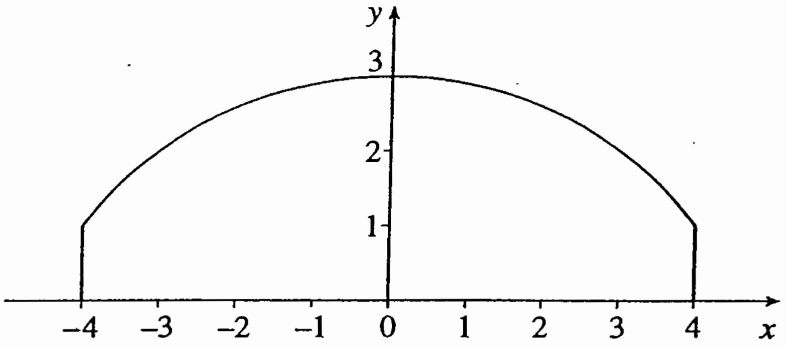

Fig. 11

(i) Use the trapezium rule with the data given in the table to obtain an estimate, A_T, of the required area.

Using Fig. 11.1 on the **insert**, shade the region corresponding to this estimate. [4]

(ii) One of the team points out that the quadratic curve C, with equation

$$y = 2\left(1 - \left(\tfrac{1}{4}x\right)^2\right) + 1,$$

passes through three of the data points and is otherwise always below the curve representing the roof of the tunnel. Draw the curve C on Fig. 11.2 on the **insert**.

The region bounded by the curve C, the x-axis and the lines $x = \pm 4$ is used to give a second estimate, A_C, for the area of the cross-section of the tunnel.

Find the exact value of A_C. State with reasons which is the better estimate, A_T or A_C. [6]

(iii) Another member of the team had estimated the area as a sum of rectangles, A_R, where

$$A_R = 2 \times [(1 \times 3.00) + (1 \times 2.90) + (1 \times 2.59) + (1 \times 2.00)] = 20.98.$$

Use Fig. 11.3 on the **insert** to show the area she found.

Show that it is now possible to give the area of the cross-section in the form $(A \pm e)$ yards^2, stating the smallest justifiable value of e and the corresponding value of A.

Explain, giving your reasons, how you think this value of A compares with the true value of the area of the cross-section. [6]

[Total:16]

Candidate Name	Centre Number	Candidate Number

Oxford, Cambridge and RSA Examinations

General Certificate of Education
Advanced Supplementary (AS) and Advanced Level

former Oxford and Cambridge modular syllabus

MEI STRUCTURED MATHEMATICS 2601

Pure Mathematics 1

Tuesday **5 JUNE 2001** Moming 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

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Write your name, Centre number and candidate number in the spaces provided at the top of this page.

HN/2

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Insert for Question 11

11 (i)

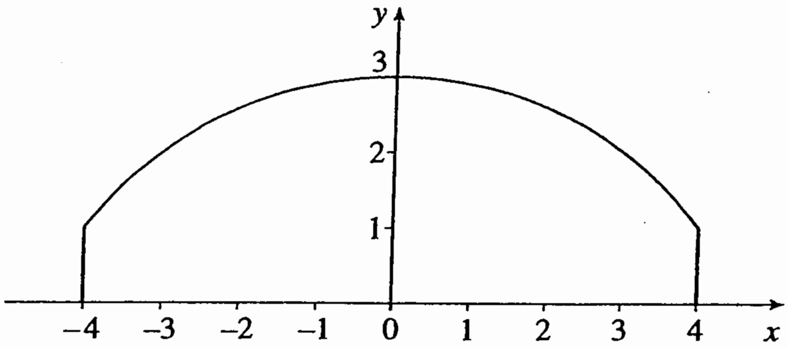


Fig. 11.1

(ii)

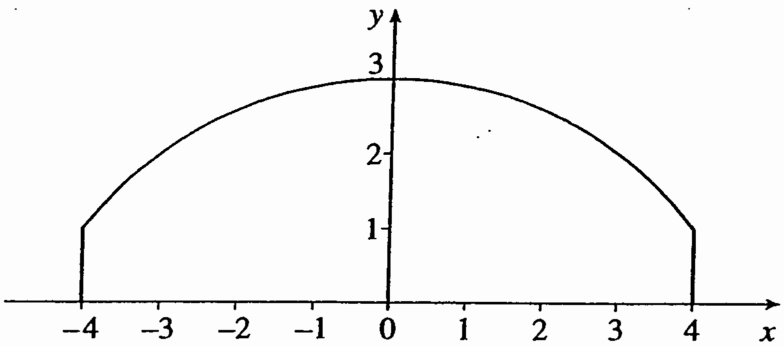


Fig. 11.2

(iii)

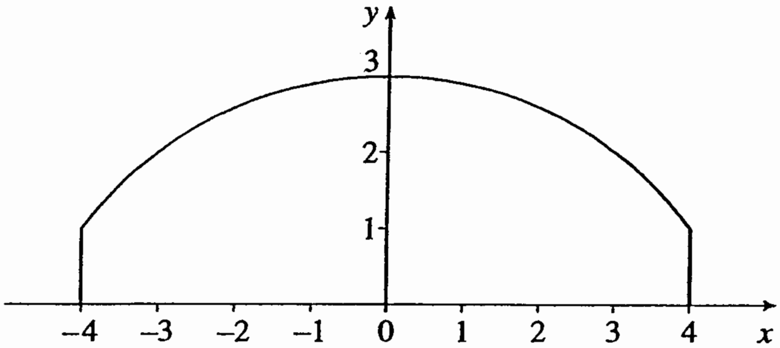
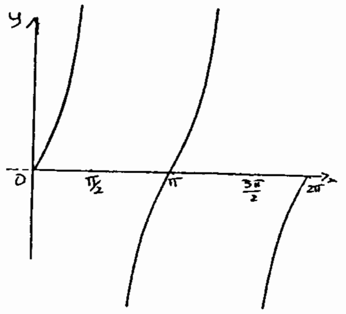


Fig. 11.3

Mark Scheme

SECTION A

1.	$2x^6 + c$	B1 + B1		2
2.**	210°, 330°	B3	B2 for 1 soln -1 if extras between 0 and 360°, ignore extras outside range. If B0, then M1 for $\sin\theta = -0.5$	3
3.**	$-3 < x < 5$	B3	1 for $x < 5$ 2 for $x > -3$ } -1 once for < etc SC2 for $-3 < \|x\| < 5$ After 0, M1 for $-4 < x - 1 < 4$	3
4.**	3360 condone $3360x^6$	B3	2^4 B1 ${}^{10}C_4$ or ${}^{10}C_6$ or equiv. or better B1	3
5.	$5^2 - 4p^2$ $5^2 - 4p^2 >\geq 0$ $p^2 \leq< 25/4$ or equivalent, cao, except condone $p^2 < 25/4$ if 2nd M1 already lost	M1 M1 M1		3
6.	$(x+3)^2 - 8$ $-3 \pm \sqrt{8}$ or $-0.17(1\ldots)$ or -0.2 and $-5.8(2\ldots)$ and -5.82843	B1 B1 B2 or B1 + B1	B1 for $a = 3$, B1 for $b = -8$ M1 for $x + 3 = \pm\sqrt{8}$ or $\dfrac{-6 \pm \sqrt{36-4}}{2}$	4
7.	30° $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ or $\sqrt{\frac{1}{3}}$ 	B1 B1 G2	B1 for $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$ correct or for all correct but θ axis not scaled. Condone x instead of θ or axis scaled in degrees	4
8.	$(x-1), (x-3)$ 3rd factor $(x+b)$ $b = -8$	B1, B1, B1 B1		4
9.**	$\frac{dy}{dx} = 3x^2 + 2x + k$ $k = -5$ $x = -\frac{5}{3}$ isw or -1.66 to -1.67	B1 B1 B2	B1 for other ans. in range -1.6 - -1.7 or for $(3x+5)(x-1)$ seen or their k subst correctly in quad. formula or factors ft their k or $\frac{dy}{dx}$ quadratic	4
			Total Section A	30

SECTION B

10.**	(i) $\frac{dy}{dx}=\frac{1}{2}x-1$	M1		
	Grad at A = $\frac{1}{2}$	A1	Must see method	
	Tgt is $y - -\frac{3}{4} = \frac{1}{2}(x-3)$	M1	Or shows $\left(3,\frac{-3}{4}\right)$ fits line B1	
	Completion (NB answer given)	A1	Or shows $\left(3,\frac{-3}{4}\right)$ fits curve B1	
	OR			
	$\frac{1}{4}x^2 - x = \frac{1}{2}x - \frac{9}{4}$	M1		
	$k(x^2 - 6x + 9) = 0$	A1		
	$(x-3)^2 = 0$ or $b^2 - 4ac = 0$	M1		
	equal roots, so tgt [at $x = 3$]	A1	Needs '3, 3' or 'equal roots' or 'one solution' so tgt	4
	(ii) grad of BC = –2 seen	B1		
	$\frac{1}{2}x - 1 = -2$	M1	ft their grad of BC	
	$x = -2$ ft	A1 ft		4
	B is (–2, 3) cao	A1		
	tgt at B is $y - 3 = -2(x+2)$	M1	ft for their –2 and (-2,3)	
	Solving eqns $y = -2x - 1$ and $y = \frac{1}{2}x - \frac{9}{4}$	M1	ft their tgt at B	
	C is $(\frac{1}{2}, -2)$ cao	A1		3
	$AC^2 = 2.5^2 + 1.25^2$ [= $\frac{125}{16}$]	M1 ft	or $BC^2 = 2.5^2 + 5^2$ [= $\frac{125}{4}$] ft if correct Pythagoras seen using their C and / or B	
	0.5 × their AC ×their BC	M1		
	completion - all correct	A1	NB answer $\frac{125}{16}$ given	
	OR: trapezium – 2 triangles:		A0 if rounded numbers used	
	$\frac{1}{2}\times[5+\frac{5}{4}]\times 5 - \frac{1}{2}\times\frac{5}{2}\times 5 - \frac{1}{2}\times\frac{5}{2}\times\frac{5}{4} = \frac{125}{16}$		M1 trapezium, M1 both triangles, A1 all correct	3
11.**	(i) Attempt to use trapezium rule	M1	eg with one slip	
	3+1+2(2.9+2.59+2) seen with any height	A1	or [1+1+2(2+2.59+2.9 +3+2.9+2.59+2)] with any ht	
	$A_T = 18.98$	A1	$A_T = 18.98$ B3, 18.36, 9.49,	
	Correct lines and shading on graph	D1	37.96 B2, 16 B1	4
	(ii) curve through (–4, 1)(0, 3)(4, 1) and below curve elsewhere	D1		
	attempt at integration	M1		
	$3x - \frac{x^3}{24}$	A1		
	$A_C = 18.\dot{6}$	A2	condone 18.67 or better; A1 for worse ans 18.6 – 18.7	
	both underestimates so larger is better	R1	ft larger of their A_T and A_C	6
	(iii) correct rects. drawn above curve	D1	ignore any shading	
	area lies between 20.98 and larger of their A_T and A_C	B1ft	ft if both lower than 20.98	
	$e = 1$	B1		
	$A = 19.98$	B1		
	Rects further away from curve than traps	M1	accept 'A_T is better than A_R' or equiv ft	
	so overestimate	A1		6
			Total Section B	30
			Total for paper	60

Examiner’s Report

Pure Mathematics 1 (2601)

General Comments

The marks gained covered the whole range from 0 to 60. There were relatively few scripts in the upper 50s, but nonetheless evidence of a good grasp of the specification by the better candidates. Sadly, this was not so for the candidates who scored less than 10 – who were fairly common in some centres. The evidence here was that they had not done sufficient revision to enable them to feel confident on some sections of the specification. Their workload in other subjects in this initial AS year will doubtless have contributed to this. Many candidates were clearly not familiar with the new formula booklet and did not know the quadratic formula, nor did they quote the formulae relevant to Pure 1 that are in the booklet, namely the binomial expansion and the trapezium rule. Centres should note that there was occasional suspicion that not all candidates were using scientific calculators. However, since method was required, such candidates did not receive credit for their answers, for instance to the integral in question 11.

Comments on Individual Questions

Question 1 (Integration)

Most candidates scored 1 or more here, although many forgot '$+c$'. However, the weakest candidates often demonstrated confusion even here, with differentiation or wrong integration attempted.

$2x^6 + c$

Question 2 (Trigonometric Equation)

Credit was given for some attempt at solution such as obtaining $\sin x = -0.5$, but some candidates produced only sketch graphs or tried to solve $\sin x = 0.5$, omitting the negative sign. Better candidates often gained full marks.

$210°, 330°$

Question 3 (Inequation)

Weaker candidates were often not happy with the modulus function. Many ignored it, producing only the solution $x < 5$, or worse. Better candidates often gained full marks.

$-3 < x < 5$

Question 4 (Binomial)

Sadly, some candidates attempted to write out the whole expansion of $(2 + x)^{10}$. Similarly some wrote out all of Pascal's triangle up to this row, sometimes with errors. Better candidates gave only the required term and often gained full marks.

3360

Question 5 (Roots of a quadratic)

Few candidates gained the full 3 marks here, with many candidates attempting only numerical substitution rather than algebraic argument.

[answer given]

Question 6 (Completing the square)

Many candidates gained the first 2 marks, but some thought –3 and 8 were the solutions to the equation, rather than using the completed square. Many candidates found the correct solutions using the quadratic formula, but incorrect versions of the formula were also seen.

$(x + 3)^2 - 8 \; -3 \pm \sqrt{8}$ or 0.17157 and –5.82843

Question 7 (Radians)

The first part was usually correct, but calculator answers were often given as exact answers. Plenty of good sketches of the graph were seen, but also plenty of incorrect ones.

$30°, \frac{1}{\sqrt{3}}$, sketch graph

Question 8 (Factor theorem)

Many obtained the factors $(x - 1)$ and $(x - 3)$ but found the third factor beyond them, although fully correct answers were frequent from the better candidates. Some tried working with simultaneous equations to find the coefficients rather than using the remainder theorem – rarely successfully, and very time-consuming.

$(x - 1)(x - 3)(x - 8)$.

Question 9 (Minimum)

Most candidates were able to do the differentiation correctly and many of these went on to find k. Fewer went on to find the value of x for the maximum, but this was one of the better attempted questions in section A, with many earning full marks.

$3x^2 + 2x + k,\ k = -5,\ x = -\frac{5}{3}$.

Question 10 (Tangents)

(i) Many candidates completed only half the task, for instance in showing that the line and the tangent had the same gradient, or that both went through $(3, -\frac{3}{4})$. However, fully correct solutions were also frequent. One common error was to find the gradient of the curve as $\frac{1}{2}x$

(ii) Many good solutions were seen, but many candidates, having found the gradient of the tangent at B as –2, then used the normal at A as the tangent at B. Follow-through method marks were awarded where appropriate, for instance for attempts to solve linear simultaneous equations to find C, or finding the distance between two points.

Question 11 (Area under a curve)

(i) Although there were the usual errors in applying the trapezium rule, many candidates applied it correctly and gained full marks in this part.

(ii) Although most attempted the integral, simplifying it and integrating correctly was more difficult. Plenty of correct solutions were seen, but also plenty of errors, even by good candidates. Relatively few stated clearly that since both were underestimates, the larger was better.

(iii) The diagram was frequently correct, but few candidates appreciated that the area lay between 20.98 and the larger of their A_T and A_C, nor used the relative accuracy of the two in their final comment. Only a few candidates scored full marks in this last part. Weaker candidates who wasted time in question 4, for example, were also running out of time by this point.

(i) $A_T = 18.98$ (ii) $A_C = 18.\dot{6}$, (iii) $e = 1, A = 19.98$